

# THE LOGICAL STRUCTURE OF PRINCIPLES IN ROBERT ALEXY. A CRITICAL ANALYSIS

Juan Pablo ALONSO<sup>1</sup>

## 1. INTRODUCTION

My purpose in this work is to explore the logical structure of principles proposed by Robert Alexy, in particular their structure as optimization commands.

## 2. THE LOGICAL STRUCTURE OF PRINCIPLES IN ROBERT ALEXY

In “Deber Ideal”<sup>2</sup> Alexy describes his proposal on the logical structure of principles as derived from the logical structure of norms. Alexy starts from what today could be labeled as standard logic of norms (that which accepts the classic deontic modalities of obligation -also “duty” or “command”-, prohibition and permission<sup>3 4</sup>). In Alexy’s view, rules express real or definitive commands; principles, on the other hand, express ideal or *prima facie* requirements, or “pro-tanto mandates”: the command of principles applies once other opposed considerations are discarded<sup>5</sup>.

As to the logical structure of principles, Alexy presents it as a derivation of the deontic modality “Obligatory”, to which he adds one aspect: optimization.

While a rule of obligation imposes a plain and simple duty to do p (“Op”), a principle, according to Alexy, imposes the obligation to optimize p (“O Opt p”). For Alexy, in this logical structure, “p” “would be an empirical object of optimization, this is, not a normative one”<sup>6</sup>.

Alexy holds that, alternatively, an optimization mandate could be reconstructed so that the object to optimize is not empirical but normative. The structure in this case would be “O Opt Op”. Following the author, this formulation is the counterpart of the “Ideal Obligation” (“Oip”). Between both formulations there is a relation of mutual implication, in so far as “the optimization commands and the ideal obligation are two sides of the same thing”<sup>7</sup>:

$$(1) O \text{ Opt Op} \leftrightarrow Oip$$

---

<sup>1</sup> University of Buenos Aires. [jalonso@derecho.uba.ar](mailto:jalonso@derecho.uba.ar)

<sup>2</sup> Robert Alexy, “Deber Ideal” in *La construcción de los Derechos Fundamentales*. Ad-Hoc, Buenos Aires, 2010 (Spanish translation by Laura Clérico and Jan Sieckmann).

<sup>3</sup> “Deber Ideal”, p. 42.

<sup>4</sup> A description of this standard logic of norms may be found in Teresa Echave, María Eugenia Urquijo and Ricardo Guibourg. *Lógica, proposición y norma*, Astrea, Buenos Aires, 1995 (pp. 118 ff).

<sup>5</sup> “Deber Ideal”, p. 43.

<sup>6</sup> “Deber Ideal”, p. 45.

<sup>7</sup> “Deber Ideal”, p. 47.

On the contrary, between the first formulation and the ideal obligation there is a relation of simple implication, because the existence of an optimization command is a sufficient condition for the existence of an ideal obligation<sup>8</sup>:

$$(2) O \text{ Opt } p \rightarrow Oip$$

Given the mutual implication of the formula “ $O \text{ Opt } Op \leftrightarrow Oip$ ”, then the following implication is also valid for Alexy:

$$(3) O \text{ Opt } p \rightarrow O \text{ Opt } Op \quad [\text{by hypothetical syllogism in (2) and (1)}]$$

This Alexyan logic of principles is complemented by predicate logic and quantifiers<sup>9</sup>, therefore the complete logical structure of principles for Alexy derives from that of conditional norms, adding “Opt” to the consequent, and hence the following ideal deontic modalities are obtained: “Ideal Obligation ( $Oip$ )”, “Ideal Permission” ( $Pip$ ) and Ideal Prohibition ( $\neg Pip$ ). Lets see two examples proposed by Alexy for this structure.

$$(x) (T_1 x \rightarrow P_i R x)$$

[For every  $x$  ( $x$ ), if  $x$  is an expression of an opinion ( $T_1$ ), then ( $\rightarrow$ ) it is *prima facie* permitted ( $P_i$ ), to do  $x$  ( $R$ )]<sup>10</sup>

$$(x) (T_2 x \rightarrow \neg P_i R x)$$

[For every  $x$  ( $x$ ), whenever  $x$  is a restriction for the right to personality ( $T_2$ ), then ( $\rightarrow$ ) it is not permitted ( $P_i$ ), to do  $x$  ( $R$ )]<sup>11</sup>

My aim is to raise some questions on the proposal of Alexy, in particular around the logical functions attributed to “Opt” within the general logical structure. Specifically, I will analyze whether “Opt” should be understood (i) as part of the modalization, (ii) as part of the consequent, or (iii) as an independent element. The questions laid on this element “Opt” will refer to the most basic formula “ $O \text{ Opt } p$ ”, but they could be applied to the more complex formula “ $O \text{ Opt } Op$ ”.

Likewise, I will make some questions on how the notions of ideal permission ( $Pip$ ) and ideal prohibition ( $\neg Pip$ ) can be derived from the basic form “ $O \text{ Opt } p$ ”.

Finally, some other questions rise on the conditional structure proposed by Alexy for principles.

---

<sup>8</sup> “Deber Ideal”, p. 55

<sup>9</sup> This combination (standard deontic modalizations + predicate logic + quantifiers) was already used by Alexy in his *Teoría de la argumentación jurídica* (Centro de Estudios Constitucionales, Madrid 1989, Spanish translation by Manuel Atienza and Isabel Espejo), pp. 214 ff.

<sup>10</sup> “Deber Ideal”, p. 50.

<sup>11</sup> “Deber Ideal”, pp. 50-51.

### 3. WHAT IS THE LOGICAL FUNCTION OF “OPT”?

I will analyze initially the following three possible hypotheses: (i) “Opt” is an independent element; (ii) “Opt” is part of the modalized action (or state of affairs); (iii) “Opt” is part of the deontic modalizer.

1)	2)	3)
O    Opt    p	O    Opt p	O Opt    p
¬O   Opt    p	¬O   Opt p	¬O Opt    p
O    ¬Opt    p	O    ¬Opt p	O Opt    ¬p
¬O   ¬Opt    p	¬O   ¬Opt p	¬O Opt    ¬p
O    Opt    ¬p		
¬O   Opt    ¬p		
O    ¬Opt    ¬p		
¬O   ¬Opt    ¬p		

Although Alexy does not accept expressly any of the hypotheses above, it seems that, since he accepts the implication “ $O \text{ Opt } p \rightarrow O \text{ Opt } Op$ ”, option 3 would be correct. This is so because Alexy states that his logic of principles derives from deontic logic and some models of deontic logic do accept the theorem “ $Op \rightarrow O Op$ ”<sup>12</sup>. Therefore, in this case it would seem that Alexy holds that “Opt” is part of the deontic modalization, because otherwise the theorem would be “ $O \text{ Opt } p \rightarrow O O \text{ Opt } p$ ” if “Opt” was part of the modalized action or state of affairs, and Alexy does not hold this theorem.

From a different point of view, very intuitive, it would seem that hypothesis 2 is correct, given that optimizing (“Opt”) is an action that, as any other action, could be subject to modalization (normativization).

However, this reasoning would also imply the viability of hypothesis 1, because both actions (optimizing and p) could be both combined with their action and with their omission, i.e., it could be a norm (specifically a principle) making obligatory to optimize the welfare of the less favored citizens (Opt p); other determining that it is obligatory not to optimize the welfare of wealthy citizens (¬Opt p) and, finally, a third one whereby it is obligatory to optimize the not-welfare of those who breached the most important rules of life in society (Opt ¬p). This option 1 entails the consequence that the basic normativization forms would no longer be four<sup>13</sup> but eight, and that does not seem as held by Alexy, since he proposed three ideal forms [ideal obligation ( $O;p$ ), ideal prohibition ( $\neg P;p$ ) and ideal permission ( $P;p$ )] and it could be assumed that Alexy would accept the fourth [ideal permission to omit ( $P;\neg p$ )].

On the grounds of the considerations above, my first question is: what is the logical function of element “Opt”? Is it part of the deontic modalization, of the modalized action (state of affairs) or is an independent element?

<sup>12</sup> For instance, the Deontic System S4 proposed by Pablo Navarro and Jorge Rodríguez in *Deontic Logic and Legal Systems*, Cambridge University Press, New York, 2014, p. 31.

<sup>13</sup> The four basic forms of normativization are obligation ( $Op$ ), prohibition ( $Vp \equiv O\neg p$ ), permission to do ( $Pp \equiv \neg O\neg p$ ), and permission to omit ( $P\neg p \equiv \neg Op$ ). Please refer to section 4 below.

#### 4. WHICH ARE THE RULES OF TRANSFORMATION AND INFERENCE OF THE LOGIC OF PRINCIPLES PROPOSED BY ALEXY?

Alexy notes that his logic of principles is a part of deontic logic; one of the features of the latter is the existence of four basic modalizations, mutually inter-definable using the deontic operators Obligatory (O), Permitted (P) and Prohibited (V)<sup>14</sup>:

Obligatory p:	Op	≡	¬P¬p	≡	V¬p
Prohibited p:	O¬p	≡	¬Pp	≡	Vp
Permitted p:	¬O¬p	≡	Pp	≡	¬Vp
Permitted ¬p:	¬Op	≡	P¬p	≡	¬V¬p

In “Deber ideal”, Alexy shows how to pass from simple Obligation to p (Op) to the Obligation to optimize p (O Opt p), from there to the ideal obligation by implication (Oip) and from there to the obligation to optimize the norm “Op” (O Opt Op), by mutual implication (bi-conditional).

The author also uses the notions of Ideal Permission (P<sub>i</sub>p is his notation) and Ideal Prohibition (¬P<sub>i</sub>p in his notation). He does not explain, however, how the inter-definability between these ideal deontic modalizers works, not either how to pass from the logical forms of deontic logic to the ideal permission or ideal prohibition.

To determine how inter-definability works it is necessary to provide previously an answer to the first question above, because if optimization “Opt” is an independent element of the modalizer and of the modalized action or state of affairs (hypothesis 1 of section III above), then we would not have four basic forms anymore, but eight (unless any (or some) of the eight forms should be eliminated under any given criterion].

Nevertheless, even choosing any of the hypotheses in the previous section over the others, there would still be doubts as to the sequence (rules of transformation and inference) that should be followed to reach to the Ideal Permission and Ideal Prohibition. There are several options, as follows:

Option a): Keeping hypothesis 3) above and follow the inter-definability rules of deontic logic.

O Opt p	→	O <sub>i</sub> p	(Ideal Obligation to p)
O Opt ¬p	→	V <sub>i</sub> p	(Ideal Prohibition to p)
¬O Opt p	→	P <sub>i</sub> ¬p	(Ideal Permission to not p)
¬O Opt ¬p	→	P <sub>i</sub> p	(Ideal Permission to p)

Option b): Use hypothesis 2) above, and follow the inter-definability rules of deontic logic.

O Opt p	→	O <sub>i</sub> p	(Ideal Obligation to p)
O ¬Opt p	→	V <sub>i</sub> p	(Ideal Prohibition to p)
¬O Opt p	→	P <sub>i</sub> ¬p	(Ideal Permission to not p)
¬O ¬Opt p	→	P <sub>i</sub> p	(Ideal Permission to p)

<sup>14</sup> *Lógica, proposición y norma*, p. 123.

It is clear that the choice for one model or another does not affect the Ideal Obligation or Duty, not the Ideal Permission to omit either (they remain equivalent). Albeit, in both options the content of Ideal Prohibition and Ideal Permission changes substantially. Indeed, as far as Ideal Prohibition is concerned, option a) establishes something as “it is obligatory to optimize the not-welfare of those who committed murder” while option b) would establish something as “it is obligatory not to optimize the welfare of wealthy citizens”; As far as the Ideal Permission is concerned, the results are similar.

We could even propose other options such as:

Option c): using the Alexian notion of normative optimization (not factual), creating new inter-definability rules<sup>15</sup>:

O	Opt	O	p	↔	O <sub>i</sub>	p	(Ideal Obligation to de p)
O	Opt	V	p	↔	V <sub>i</sub>	p	(Ideal Prohibition to p)
O	Opt	P	¬p	↔	P <sub>i</sub>	¬p	(Ideal Permission to not p)
O	Opt	P	p	↔	P <sub>i</sub>	p	(Ideal Permission to p)

There would be more options, although I understand that it is senseless to keep exploring this path.

On the grounds of the considerations above, my second question would be: does the inter-definability of deontic logic apply to the logic of principles of Alexy? If so, which would be the transformation and inference rules to pass from the Prohibition to p (O¬p) to the Ideal Prohibition to p (¬P<sub>i</sub>p) and from the Permission to p (¬O¬p) to the Ideal Permission to p (P<sub>i</sub>p)?

## 5. REGARDING THE CONDITIONAL STRUCTURE OF PRINCIPLES FOR ALEXY, DOES THE WEAKENING OF THE CONSEQUENT PROPOSED BY ALEXY IMPLY THE STRENGTHENING OF THE ANTECEDENT OR THIS MAY BE WEAKENED AS WELL?

Many legal philosophers hold that the logical structure of principles is analogue to a certain extent to the logical structure of conditional norms, for instance Alexy, Atienza and Ruiz Manero or Alchourrón and Bulygin<sup>16</sup>. Other scholars claim that the logical structure of logic should be reconstructed with the schemes of preference logic<sup>17</sup> or other semantic structures, but they are not the majority.

Those holding the structural analogy between principles and conditional norms usually claim also that principles are a “weakened” version of norms. In this regard, there are three possible options: (i) weakening the antecedent, (ii) weakening the consequent; and (iii) weakening the connective between them. Atienza and Ruiz Manero exemplify option (i), Alexy option (ii) and Alchourrón options (iii) and (i).

Atienza and Ruiz Manero<sup>18</sup> propose an elegant scheme to separate the different types of principles and rules, as follows:

<sup>15</sup> This option was suggested by Hugo Zuleta.

<sup>16</sup> *Sistemas normativos*, Carlos Alchourrón y Eugenio Bulygin. Astrea, Buenos Aires, 2012, pp. 118 ff.

<sup>17</sup> See Navarro and Rodríguez, *Deontic Logic and Legal Systems*, and Juan Pablo Alonso, “Principios implícitos y coherencia”, *DOXA* N° 36, Alicante, 2013.

<sup>18</sup> *Las Piezas del Derecho*, Manuel Atienza y Juan Ruiz Manero. Ariel Derecho. Barcelona, 1996.

		Antecedent (case)	Consequent (solution)
Rules	Action rules	Closed	Closed
	End rules	Closed	Open
Principles	Strict sense	Open	Closed
	Directives	Open	Open

As can be noticed, for Atienza and Ruiz Manero the central feature of the genus “principles” is that the case is open, this is, subject to further precisions. This is what differences the genus “principles” from the genus “rules” (whose antecedent or case is closed). Clearly, the Spanish authors choose to weaken the antecedent of the conditional, holding also that for principles in the strict sense, the solution is closed; once the case is defined or specified, the solution is of the type “Op” or “¬Pp”, this is, a logic form of standard deontic logic, not modified<sup>19</sup>.

We will see that Alexy’s proposal seems to be opposed to the one of Atienza and Ruiz Manero, since for Alexy, the case of principles in the strict sense is closed and the solution is open (or “weakened”, or “prima facie”).

As to Alchourrón, while he did not stated expressly his views on the structure of principles, his thesis on defeasibility of norms are, in my opinion, applicable to the issues at hand<sup>20</sup>.

Alchourrón analyses different proposal to weaken the classic conditional connective (in any of its versions -material implication, generalized conditional, etc.-) and replace it with a connective that does not satisfy the law of strengthening the antecedent and Modus Ponens. For instance:

$$p > Oq$$

As Alchourrón analyzes, the problem of weakening the connective lies in the loss of inferential capacity. In other words, a connective that does not satisfy the strengthening of the antecedent and Modus Ponens is not useful to justify deductively any practical decision, i.e., a judicial sentence.

Additionally, Alchourrón claims that those using defeasible conditionals also hide the weakening of the antecedent in the common conditional. This is, the use of defeasible conditionals mistakenly transfers to the connective a problem belonging to the antecedent of the conditional.

Hence, Alchourrón proposes to abandon the use of defeasible conditionals (also called “*prima facie* conditionals”), to keep a strong connective (Alchourrón proposes the generalized conditional “ $\Rightarrow$ ”) that satisfies the strengthening of the antecedent and Deontic Modus Ponens, adding a revision operator to the antecedent. The formula is as follows:

$$f(p) \Rightarrow Oq$$

<sup>19</sup> I should clarify that I share some of the criticisms made by Giovanni Ratti in “Una nota sobre la forma lógica de los principios jurídicos”, in *El gobierno de las normas* (Marcial Pons, Barcelona, 2013) to the proposal of Atienza and Ruiz Manero; I cannot elaborate further on the matter in this work.

<sup>20</sup> Carlos Alchourrón, “Condicionalidad y la representación de las normas jurídicas”, in Carlos Alchourrón and Eugenio Bulygin, *Análisis lógico y Derecho*, Centro de Estudios Constitucionales, Madrid, 1991.

[If p happens, and no circumstances arise that may be causes to revise p -f(p)-, then (strong conditional  $\Rightarrow$ ) Obligatory q (Oq)].

The revision is a function affecting the antecedent of the conditional, and that selects a certain subset of p cases, the most usual or those where certain exceptions are not verified.

In my view, the proposal of Alchourrón is the best theory for the claim on the structural analogy between principles and conditional norms. In any event, the question on whether there is a best reconstruction of the logical structure of principles that keeps no analogies with the logical structure of conditional norms (i.e., a structure as in preference logic) remains open though.

Professor Alexy seems to hold a position inverse to that of Alchourrón, and of Atienza and Ruiz Manero. In “Deber ideal” the logical structure of his principles is the following:

- (x)  $(T_1 x \rightarrow P_i R x)$
- (x)  $(T_2 x \rightarrow \neg P_i R x)$

As may be noticed, the connective used by Alexy is material implication ( $\rightarrow$ ), connective that satisfies the strengthening of the antecedent and Deontic Modus Ponens. The antecedent, on the other hand, lacks revision functions or any other weakening mechanisms. Weakening, apparently, affects only to the consequent.

But the weakened consequent proposed by Alexy keeps, in my view, those problems suggested in my two previous questions.

On the grounds of the considerations above, my third question to Alexy is the following: Is my description correct as to the logical structure of principles as suggested by Alexy –meaning that the antecedent and the connective are strong and the only component weakened is the consequent -? Does the proposal of Alexy entail the rejection of those proposals that weaken the antecedent of the conditional?

## **6. CONCLUSIONS: THREE QUESTIONS TO PROFESSOR ROBERT ALEXY**

**I.** What is the logical function of element “Opt”? Is it part of the deontic modalization, of the modalized action (state of affairs) or is an independent element?

**II.** Does the inter-definability of deontic logic apply to the logic of principles of Alexy? If so, which would be the transformation and inference rules to pass from the Prohibition to p (O¬p) to the Ideal Prohibition to p (¬Pip) and from the Permission to p (¬O¬p) to the Ideal Permission to p (Pip)?

**III.** Is my description correct as to the logical structure of principles as suggested by Alexy -meaning that the antecedent and the connective are strong and the only component weakened is the consequent-? Does the proposal of Alexy entail the rejection of those proposals that weaken the antecedent of the conditional?